

Qubit rotation in QHE

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Abstract

In Quantum Hall effect the ground state wave function at $\nu = 1$ is the building block of all other states at different filling factors. It is developed by the entanglement of two spinors forming a singlet state. The inherent frustration visualized by the non-abelian matrix Berry phase is responsible for the quantum pumped charge to flow in the Hall surface. The Physics behind the Quantum Hall states is studied here from the view point of topological quantum computation.

Key words: spin echo, Berry phase.

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1 Introduction

Entanglement is one of the basic aspects of quantum mechanics. It was known long ago that quantum mechanics exhibits very peculiar correlations between two physically distant parts of the total system. Afterwards, the discovery of Bell's inequality (BI) [1] showed that BI can be violated by quantum mechanics but has to be satisfied by all local realistic theories. The violation of BI demonstrates the presence of entanglement [2]. The theorem of BI may be interpreted as incompatibility of requirement of locality with the statistical predictions of quantum mechanics. So to study the Bell state, the role of *local* spatial observations, apart from spin correlations, should also be taken into account [3]. This indicates that the spatial variation of a quantum mechanical state would carry its memory through some geometric phase known as Berry phase(BP) [4]. It is expected that the influence of BP on an entangled state could be linked up with the local observations of spins.

To have a comprehensive view of the quantum mechanical correlation between two spin $1/2$ particles in an entangled state, we should take into account the role of the Berry phase related to a spinor. This study belong to the field of geometric quantum computation where noticeably the geometrical and topological gates are resistant to local disturbances. In quantum mechanical entanglement of two spin $1/2$ particles the Berry phase plays an important role during spin echo method [5]. Kitaev described [6] the topological and quantum computer as a device in which quantum numbers carried by quasiparticles residing in two dimensional electron gas have long range Aharonov-Bohm (AB) interactions between one another. These AB interactions are responsible for nontrivial phase values during interwinding of quasiparticles trajectories in course of time evolution of qubits in Quantum Hall Effects (QHE). Quantization plays an important role to realize the Physics behind different states of QHE from the view point of Berry phase [7]. Recently we have studied the rotation of a quantized spinor identified as qubit in presence of magnetic field under the spin echo method [8]. We will aim at understanding the rotation of QHE qubits specially in the lowest Landau level $\nu = 1$ and then parent states $\nu = 1/m$ from the view point of geometric quantum computation.

2 Quantization of Fermi field and qubits of singlet states

The quantization of Fermi field can be achieved assuming anisotropy in the internal space through the introduction of direction vector as an internal variable at each space-time point [9]. The opposite orientations of the direction vector correspond to particle and antiparticle. Incorporation of spinorial variables $\theta(\bar{\theta})$ in the coordinate result the enlargement of manifold from S^2 to S^3 . This helps us to consider a relativistic quantum particle as an extended one, where the extension involves gauge degrees of freedom. As a result

the position and momentum variables of a quantized particle becomes

$$Q_\mu = i \left(\frac{\partial}{\partial p_\mu} + A_\mu \right), \quad P_\mu = i \left(\frac{\partial}{\partial q_\mu} + \tilde{A}_\mu \right) \quad (1)$$

where q_μ and p_μ are related to the position and momentum coordinates in the sharp point limit and $A_\mu(\tilde{A}_\mu)$ are non-Abelian matrix valued gauge fields belonging to the group $SL(2C)$.

In three space dimension, in an axis-symmetric system where the anisotropy is introduced along a particular direction, the components of the linear momentum satisfy a commutation relation of the form [10]

$$[p_i, p_j] = i\mu\epsilon_{ijk} \frac{x^k}{r^3} \quad (2)$$

Here μ corresponds to the measure of anisotropy and behaves like the strength of a magnetic monopole. Indeed in this anisotropic space the conserved angular momentum is given by

$$\vec{J} = \vec{r} \times \vec{p} - \mu \hat{r} \quad (3)$$

with $\mu = 0, \pm 1/2, \pm 1, \dots$. This corresponds to the motion of a charged particle in the field of a magnetic monopole. For the specific case of $l = 1/2, |m| = |\mu| = 1/2$ for half orbital/spin angular momentum, we can construct from the spherical harmonics $Y_l^{m,\mu}$, the instantaneous eigenstates $|\uparrow, t\rangle$, representing the two component up-spinor as

$$\begin{aligned} |\uparrow, t\rangle &= \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} Y_{1/2}^{1/2, 1/2} \\ Y_{1/2}^{-1/2, 1/2} \end{pmatrix} \\ &= \begin{pmatrix} \sin \frac{\theta}{2} \exp i(\phi - \chi)/2 \\ \cos \frac{\theta}{2} \exp -i(\phi + \chi)/2 \end{pmatrix} \end{aligned} \quad (4)$$

and the conjugate state is a down-spinor given by

$$|\downarrow, t\rangle = \begin{pmatrix} -Y_{1/2}^{-1/2, 1/2} \\ Y_{1/2}^{-1/2, -1/2} \end{pmatrix} = \begin{pmatrix} -\cos \frac{\theta}{2} \exp i(\phi + \chi)/2 \\ \sin \frac{\theta}{2} \exp -i(\phi - \chi)/2 \end{pmatrix} \quad (5)$$

These two spinors (up/down) represent quantized fermi field originated by an arbitrary superposition of elementary qubits $|0\rangle$ and $|1\rangle$ as for up spinor

$$|\uparrow, t\rangle = \left(\sin \frac{\theta}{2} e^{i\phi} |0\rangle + \cos \frac{\theta}{2} |1\rangle \right) e^{-i/2(\phi+\chi)} \quad (6)$$

and the down spinor becomes

$$|\downarrow, t\rangle = \left(-\cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{-i\phi} |1\rangle \right) e^{i/2(\phi+\chi)} \quad (7)$$

The states $|\uparrow, t\rangle$ and $|\downarrow, t\rangle$ can be generated by the unitary transformation matrix $U(\theta, \phi, \chi)$ [11]

$$U(\theta, \phi, \chi) = \begin{pmatrix} \sin \frac{\theta}{2} e^{i/2(\phi-\chi)} & -\cos \frac{\theta}{2} e^{i/2(\phi+\chi)} \\ \cos \frac{\theta}{2} e^{-i/2(\phi+\chi)} & \sin \frac{\theta}{2} e^{-i/2(\phi-\chi)} \end{pmatrix} \quad (8)$$

in association with the basic qubits $|0\rangle$ and $|1\rangle$

$$|\uparrow, t\rangle = U(\theta, \phi, \chi) |0\rangle, |\downarrow, t\rangle = U(\theta, \phi, \chi) |1\rangle \quad (9)$$

Over a closed path, the single quantized up spinor acquires the geometrical phase [8]

$$\gamma_{\uparrow} = i \oint \langle \uparrow, t | \nabla | \uparrow, t \rangle . d\lambda \quad (10)$$

$$= i \oint \langle 0 | U^\dagger dU | 0 \rangle . d\lambda \quad (11)$$

$$= i \oint A_{\uparrow}(\lambda) d\lambda \quad (12)$$

$$= \oint L_{eff}^{\uparrow} dt \quad (13)$$

$$= \frac{1}{2} (\oint d\chi - \cos \theta \oint d\phi) \quad (14)$$

$$= \pi(1 - \cos \theta) \quad (15)$$

representing a solid angle subtended about the quantization axis. For the conjugate state the Berry phase over the closed path becomes

$$\gamma_{\downarrow} = -\pi(1 - \cos \theta) \quad (16)$$

The fermionic or the antifermionic nature of the two spinors (up/down) can be identified by the maximum value of topological phase $\gamma_{\uparrow/\downarrow} = \pm\pi$ at an angle $\theta = \pi/2$. For $\theta = 0$ we get the minimum value of $\gamma_{\uparrow} = 0$ and at $\theta = \pi$ no extra effect of phase is realized.

It can be verified that this Berry phase remains the same if we neglect the overall phase $e^{\pm i(\phi-\chi)/2}$ from the quantized spinors as in eqs.(6) and (7) respectively. The identical value of Berry phase $\gamma_{\uparrow/\downarrow}$ in both the approaches is only possible if we consider no local frustration in the spin system otherwise the conflict between the parameters of quantized spinor will cause to have different BP [12].

In the language of quantum computation, the rotation of qubit or quantized spinor can be studied well in the background of geometric phase. Any electronic state at any instant can be written as a linear combination of the instantaneous eigenstates.

$$|\Psi(t)\rangle = c_1(t) |\Phi_1(t)\rangle + c_2(t) |\Phi_2(t)\rangle \quad (17)$$

In a cyclic change of the time period T , the instantaneous basis states $|\Phi_1(T)\rangle$ and $|\Phi_2(T)\rangle$ might return to their initial states $|\Phi_1(0)\rangle$ and $|\Phi_2(0)\rangle$ where the coefficients $c_1(T)$ and

$c_2(T)$ may not. This doubly degenerate energy level, a 2×2 matrix Berry phase Φ_c connects the final amplitudes- $c_1(T), c_2(T)$ with the initial amplitudes $c_1(0), c_2(0)$

$$\begin{pmatrix} c_1(T) \\ c_2(T) \end{pmatrix} = \Phi_c \begin{pmatrix} c_1(0) \\ c_2(0) \end{pmatrix} \quad (18)$$

Hwang et.al [13] pointed out that this non abelian matrix Berry phase is responsible for pumped charges where the charge transport in a cycle of the pump in a j th optimal channel becomes

$$Q_j = -i/2\pi \oint \langle \psi_j | d\psi_j \rangle \quad (19)$$

This charge will be only of topological in nature during transport of qubits, if the influence of dynamical phase can be eliminated. Spin echo method is a popular technic for this removal of dynamical phase where two cyclic evolutions are applied on a spinor with the second application followed by a pair of fast π transformations. Vedral et.al.[14] showed the application of spin echo to a spinor (eq.(6)).

$$\begin{aligned} |\uparrow\rangle &\xrightarrow{C_R} e^{i(\delta_\uparrow - \gamma)} |\uparrow\rangle \xrightarrow{\pi} e^{i(\delta_\uparrow - \gamma)} |\downarrow\rangle \\ &\xrightarrow{C_L} e^{i(\delta_\uparrow + \delta_\downarrow - 2\gamma)} |\downarrow\rangle \xrightarrow{\pi} e^{i(\delta_\uparrow + \delta_\downarrow - 2\gamma)} |\uparrow\rangle \end{aligned} \quad (20)$$

Here $\xrightarrow{C_R}$ introduces the dynamical and geometrical phases, δ_\uparrow and γ_\uparrow through right cyclic evolution of $|\uparrow\rangle$ spinor respectively. Similar phases of opposite orientations are developed by $\xrightarrow{C_L}$. Referring back to eqs.(15) and (16), we see that $\gamma_\uparrow = \gamma$ and $\gamma_\downarrow = -\gamma$ for $\gamma = \pi(1 - \cos\theta)$. Thus two cyclic evolutions accompanied by two π rotations eliminate the net dynamical phases doubling the geometric phase of the original state (up/down spinor) according to.

$$|\uparrow\rangle \xrightarrow{} e^{2i\gamma_\uparrow} |\uparrow\rangle, |\downarrow\rangle \xrightarrow{} e^{2i\gamma_\downarrow} |\downarrow\rangle \quad (21)$$

For two half periods of spin echo rotation we have

$$|\uparrow\rangle \xrightarrow{} e^{i\gamma_\uparrow} |\uparrow\rangle, |\downarrow\rangle \xrightarrow{} e^{i\gamma_\downarrow} |\downarrow\rangle \quad (22)$$

where the total effect of dynamical phase disappear. The spin echo method is very fruitful [15] in the construction of two qubit through rotation of one qubit (spin 1/2) in the vicinity of another. Incorporating the spin-echo for half period (as in eqn.22) we find the antisymmetric Bell's state after one cycle ($t = \tau$),

$$|\Psi_-(t = \tau)\rangle = \frac{1}{\sqrt{2}}(e^{i\gamma_\uparrow} |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - e^{-i\gamma_\uparrow} |\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \quad (23)$$

and symmetric state becomes

$$|\Psi_+(t = \tau)\rangle = \frac{1}{\sqrt{2}}(e^{-i\gamma_\uparrow} |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 + e^{i\gamma_\uparrow} |\downarrow\rangle_1 \otimes |\uparrow\rangle_2) \quad (24)$$

where $\gamma_{\downarrow} = -\gamma_{\uparrow} = -\gamma$. Splitting up these above two eqs.(23) and (24) into the symmetric and antisymmetric states and rearranging we have

$$|\Psi_{+}\rangle_{\tau} = \cos \gamma |\Psi_{+}\rangle_0 - i \sin \gamma |\Psi_{-}\rangle_0 \quad (25)$$

$$|\Psi_{-}\rangle_{\tau} = i \sin \gamma |\Psi_{+}\rangle_0 + \cos \gamma |\Psi_{-}\rangle_0 \quad (26)$$

the doublet acquiring the matrix Berry phase- Σ as rotated from $t = 0$ to $t = \tau$.

$$\begin{pmatrix} |\Psi_{+}\rangle \\ |\Psi_{-}\rangle \end{pmatrix}_{\tau} = \Sigma \begin{pmatrix} |\Psi_{+}\rangle \\ |\Psi_{-}\rangle \end{pmatrix}_0 \quad (27)$$

$$\Sigma = \begin{pmatrix} \cos \gamma & -i \sin \gamma \\ i \sin \gamma & \cos \gamma \end{pmatrix} = \cos 2\gamma \quad (28)$$

This non-abelian matrix Berry phase Σ is developed from the abelian Berry phase γ . For $\gamma = 0$ there is symmetric rotation of states, but for $\gamma = \pi$ the return is antisymmetric as the values of $\Sigma=I$ and $-I$ (where I =identity matrix) respectively.

The instantaneous quantum state can be represented by the linear combination of degenerate symmetric and antisymmetric states. Symmetric state will return to antisymmetric state over one half period of spin echo apart from a matrix valued Berry phase [16]. It may be noted that two half period rotations will complete one spin echo resulting the return of the state to itself apart from a geometrical phase factor.

$$\begin{pmatrix} |\Psi_{+}\rangle \\ |\Psi_{-}\rangle \end{pmatrix}_{2\tau} = \begin{pmatrix} \cos \gamma & -i \sin \gamma \\ i \sin \gamma & \cos \gamma \end{pmatrix} \begin{pmatrix} \cos \gamma & i \sin \gamma \\ -i \sin \gamma & \cos \gamma \end{pmatrix} \begin{pmatrix} |\Psi_{+}\rangle \\ |\Psi_{-}\rangle \end{pmatrix}_0 \quad (29)$$

Following the notion of one complete spin echo here, the state $|\Psi_{+}\rangle_{T=2\tau}$ also return to its initial state $|\Psi_{+}\rangle_0$ apart from the phase $\cos 2\gamma$.

$$\begin{pmatrix} |\Psi_{+}\rangle \\ |\Psi_{-}\rangle \end{pmatrix}_{2\tau} = \begin{pmatrix} \cos 2\gamma & 0 \\ 0 & \cos 2\gamma \end{pmatrix} \begin{pmatrix} |\Psi_{+}\rangle \\ |\Psi_{-}\rangle \end{pmatrix}_0 \quad (30)$$

In any even number of half period τ , the symmetric state will return to itself apart from Berry phase factor with increased power of $\cos 2\gamma$.

$$\begin{pmatrix} |\Psi_{+}\rangle \\ |\Psi_{-}\rangle \end{pmatrix}_{2n\tau} = \begin{pmatrix} \cos 2\gamma & 0 \\ 0 & \cos 2\gamma \end{pmatrix}^n \begin{pmatrix} |\Psi_{+}\rangle \\ |\Psi_{-}\rangle \end{pmatrix}_0 \quad (31)$$

where $n = 1, 2, 3, \dots$ are natural integers. For odd number of the half periods rotations there will be mixture of both the states. On the other hand with the value of $\gamma = \pi$, the symmetric/antisymmetric state remains same after one rotation.

In this connection we have shown recently [8] that the singlet state between two spinors at a particular instant is connected with the singlet state of elementary qubits $|0\rangle$ and $|1\rangle$ and the Berry phase of the initial antisymmetric Bell's state is $\gamma_{ent} = \pi(1 + \cos 2\theta)$

where if we introduce spin echo in the two qubit then the topological phase $\Sigma = \cos 2\gamma$ is of matrix valued.

By varying the magnetic field angle $\theta : 0 \longrightarrow \pi/3 \longrightarrow \pi/2$, the Berry phase(BP) of a qubit changes to, $\gamma : 0 \longrightarrow \pi/2 \longrightarrow \pi$, that in turn change the two qubit BP, $\Sigma : I \longrightarrow \sigma^y \longrightarrow -I$. This explains the physics behind the change from the antisymmetric Bell singlet state Ψ_- to the symmetric Bell state Ψ_+ and back to Ψ_- . We will now proceed to apply the above idea of entanglement in the field of Quantum Hall effect to study the state formation from one filling factor to another in the light of Geometric Quantum Computation.

3 Qubit formation of Quantum Hall state

Quantum Hall effect shows a prominent appearance of quantization of Hall particles involving gauge theoretic extension of coordinate by $C_\mu \in SL(2C)$ visualized by the field strength $F_{\mu\nu}$ acting as background external magnetic field. It is noted that the gauge field theoretic extension for a Fermi field associated with the direction vector ξ_μ attached to the space-time point x_μ results the field function $\phi(x_\mu, \xi_\mu)$ describing a particle moving in an anisotropic space [7].

The external magnetic field introduces frustration in the Hall system. We have considered a two-dimensional frustrated electron gas of N particles on the spherical surface of a three dimensional sphere of large radius R in a strong radial (monopole) magnetic field. In such a 3D anisotropic space we can construct the N -particle wave-function from the spherical harmonics $Y_l^{m,\mu}$ with $l = 1/2$, $|m| = |\mu| = 1/2$ (when the angular momentum in the anisotropic space is given by eq.(3)). With the description of a two component up spinor $|\uparrow\rangle = \begin{pmatrix} u \\ v \end{pmatrix}$ as in eq.(6) we can construct the N particles wave function of Hall states

$$\Psi_{N_\uparrow}^{(m)} = \prod (u_i v_j - u_j v_i)^m \quad (32)$$

for parent states $m = 1/\nu$ where ν is the Landau filling factor and this $m = J_{ij} = J_i + J_j$ is the two particle angular momentum equivalent to $m = \mu_i + \mu_j = 2\mu$ (when $i = j$). Similar manner the same Hall state with opposite polarization can be constructed by using the down spinor $|\downarrow\rangle = \begin{pmatrix} \tilde{v} \\ \tilde{u} \end{pmatrix}$

$$\Psi_{N_\downarrow}^{(m)} = \prod (\tilde{u}_i \tilde{v}_j - \tilde{u}_j \tilde{v}_i)^m \quad (33)$$

Here the two states $\Psi_{N_\uparrow}^{(m)}$ and $\Psi_{N_\downarrow}^{(m)}$ belong to the same parent filling factor but with opposite polarization of the spinors.

The above states are grouped into a family depending on the value of m . With $m = 3$ the states are the same family of the Laughlin $\nu = 1/3$ state etc. In the light of Jain [17] that regarding the filling factor the IQHE of composite fermions are the FQHE of fermions, any FQHE state can be expressed in terms of the IQHE state. It seems that

for LLL $\nu = 1$, IQHE state $\Phi_1(z)$

$$\Phi_1(z) = (u_i v_j - u_j v_i) \quad (34)$$

is the basic building block for constructing any other IQHE/FQHE state. The lowest level Hall state $\Phi_1(z)$ has a similarity with two-qubit singlet state formed by a pair of one qubit states.

There is a deep analogy between FQHE and superfluidity [18]. The ground state of anti-ferromagnetic Heisenberg model on a lattice introduce frustration giving rise to the resonating valence bond(RVB) states corresponding spin singlets where two nearest-neighbor bonds are allowed to resonate among themselves. It is suggested that RVB states [6] is a basis of fault tolerant topological quantum computation. Since these spin singlet states forming a RVB gas is equivalent to fractional quantum Hall fluid, its description through quantum computation will be of ample interest.

This resonating valence bond(RVB) where two nearest-neighbour bonds are allowed to resonate among themselves has equivalence with entangled state of two one-qubit. The antisymmetric Hall state $\Phi_1(z)$ for $\nu = 1$ is formed as one spinor at i th site rotating with Berry phase $\gamma = \pm i\pi$ in the vicinity of another at j th site captures the image of spin echo

$$|\Phi_1(z)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \quad |\downarrow\rangle_1) \begin{pmatrix} 0 & -e^{-i\pi} \\ e^{i\pi} & 0 \end{pmatrix} \begin{pmatrix} |\uparrow\rangle_2 \\ |\downarrow\rangle_2 \end{pmatrix} \quad (35)$$

$$= (|\uparrow\rangle_1 \quad |\downarrow\rangle_1) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} |\uparrow\rangle_2 \\ |\downarrow\rangle_2 \end{pmatrix} \quad (36)$$

$$= \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \quad |\downarrow\rangle_2 - |\downarrow\rangle_1 \quad |\uparrow\rangle_2) \quad (37)$$

$$= |\Psi_-\rangle \quad (38)$$

Due to symmetry, the singlet state can be written on any basis with the same form. We can rotate the spin vector by an arbitrary angle θ with the following transformation.

$$\begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix} = \begin{pmatrix} \sin \theta e^{i\phi} & \cos \theta \\ -\cos \theta & \sin \theta e^{-i\phi} \end{pmatrix} \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} \quad (39)$$

The Quantum Hall systems are so highly frustrated that the ground state $\Phi_1(z)$ is an extremely entangled state visualized by the formation of antisymmetric singlet state between a pair of i, j th spinors in the Landau filling factor ($\nu = 1$).

$$\begin{aligned} \Phi_1(z) &= \begin{pmatrix} u_i & u_j \\ v_i & v_j \end{pmatrix} = (u_i v_j - u_j v_i) \\ &= (u_i \quad v_i) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} u_j \\ v_j \end{pmatrix} \end{aligned} \quad (40)$$

We identify this two qubit singlet state as Hall qubit constructed from the up-spinor shown in the previous section

$$\Phi_1(z) = \langle \uparrow_i | \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} | \uparrow_j \rangle = \langle 0 | U_i^\dagger \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} U_j | 0 \rangle \quad (41)$$

The down spinor can construct the opposite polarization of Hall qubit

$$\Phi_1(\tilde{z}) = (\tilde{u}_i \tilde{v}_j - \tilde{u}_j \tilde{v}_i) \quad (42)$$

that has a similar representation as eq.(41)

$$\Phi_1(\tilde{z}) = \langle 1 | U^\dagger \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} U | 1 \rangle \quad (43)$$

Now these two Hall qubits of two opposite polarizations representing the state of same lowest Landau level $\nu = m = 1$ will automatically generate two respective non-abelian Berry connections. The Hall connection for up spinor becomes

$$B_\uparrow = \Phi_1(z)^* d\Phi_1(z) = \begin{pmatrix} u_i^* & v_i^* \\ u_j^* & v_j^* \end{pmatrix} \begin{pmatrix} du_i & du_j \\ dv_i & dv_j \end{pmatrix} \quad (44)$$

and similarly for down-spinor

$$\tilde{B}_\downarrow = \Phi_1(\tilde{z})^* d\Phi_1(\tilde{z}) = \begin{pmatrix} \tilde{u}_i^* & \tilde{v}_i^* \\ \tilde{u}_j^* & \tilde{v}_j^* \end{pmatrix} \begin{pmatrix} d\tilde{u}_i & d\tilde{u}_j \\ d\tilde{v}_i & d\tilde{v}_j \end{pmatrix} \quad (45)$$

The non-abelian nature of the connection or Berry phase on the Hall surface for the lowest Landau level LLL ($\nu = 1$) will remain if $i \neq j$.

$$\begin{aligned} B_\uparrow &= \begin{pmatrix} (u_i^* du_i + v_i^* dv_i) & (u_i^* du_j + v_i^* dv_j) \\ (u_j^* du_i + v_j^* dv_i) & (u_j^* du_j + v_j^* dv_j) \end{pmatrix} \\ &= \begin{pmatrix} \mu_i & \mu_{ij} \\ \mu_{ji} & \mu_j \end{pmatrix} \end{aligned} \quad (46)$$

This is visualizing the spin conflict during parallel transport leading to matrix Berry phase. In the light of Hwang et.al [13] our realization includes that in Quantum Hall effect this non-abelian matrix Berry phase is responsible for the charge flow by pumping. In this QHE matrix Berry phase

$$\gamma^H_\uparrow = \begin{pmatrix} \gamma_i & \gamma_{ij} \\ \gamma_{ji} & \gamma_j \end{pmatrix} \quad (47)$$

γ_i and γ_j are the BPs for the i th and j th spinor as seen in eq. (15) and the off-diagonal BP γ_{ij} arises due to local frustration in the spin system. Over a closed period $t = \tau$ the QHE state $\Phi_1(z)$ at $\nu = 1$ filling factor will acquire the matrix Berry phase.

$$\langle \Phi_1(z) |_\tau = e^{i\gamma^H_\uparrow} \langle \Phi_1(z) |_0 \quad (48)$$

Berry connection gets modified as the quantum state differ after one rotation. Usually when any state changes by

$$|\psi'\rangle = |\psi\rangle e^{i\Omega(c)}$$

the corresponding changed gauge becomes

$$A_{\psi'} = A_{\psi} + id\Omega(c)$$

provided $\langle\psi|\psi\rangle = 1$. We have pointed out earlier [19] that each Quantum Hall state for a particular filling factor has its distinct Berry phase. Hence BP is constant for a filling factor. The rotation shifts the BP from ground to excited level once. With these ideas we have the topological phase difference between the first excited and the ground state acquired by the rate of change of Berry phase

$$\Gamma^1 - \Gamma^0 = i \oint \langle\Phi_1(z)| d\gamma^H/d\lambda |\Phi_1(z)\rangle_0 d\lambda \quad (49)$$

The rotation of singlet state by 'n' number of turns will be

$$\langle\Phi_1(z)|^n_{\tau} = e^{in\gamma^H} \langle\Phi_1(z)|^n_0 \quad (50)$$

where $n = 1, 2, 3..$ are the natural numbers associated with the number of rotations of the singlet states. We should point out here that the antisymmetric nature of FQHE states would be visualized through the rotation of singlet states. This automatically imposes the following constraint in the topological phase

$$e^{in\gamma^H} = e^{im\pi} = -1, \quad (51)$$

$$for \quad \langle\Phi_1(z)|_{n\tau} = -\langle\Phi_1(z)|_0$$

where $m = 1, 3, 5..$ being the odd numbers to maintain the antisymmetric nature of wave function. So any number of rotations of the matrix Berry phase lead to odd multiple of π angles provided the every state remains antisymmetric. It seems that BP act as a local order parameter of QHE states.

$$\langle\Phi_1(z)|^{\frac{m\pi}{\gamma}}_{\tau} = e^{im\pi} \langle\Phi_1(z)|^{\frac{m\pi}{\gamma}}_0 \quad (52)$$

Earlier we showed [7] that the Berry phase for $\nu = 1/m$ state is $\gamma == m\pi\theta = 2\pi\mu\theta$ where θ is a coupling constant. This motivated us to write

$$\langle\Phi_1(z)|_{\tau} = e^{im\pi\theta} \langle\Phi_1(z)|_0 \quad (53)$$

This makes the experimental observation of parent state in FQHE at $m = odd(3, 5, 7)$ more transparent. It also shows that the topological phase is responsible for controlling the statistics of the Hall state. In absence of frustration, the role of matrix Berry phase is

trivial. In other words γ_{ij} becomes zero leading to diagonal matrix Berry phase provided the two particles have identical θ and ϕ values.

$$\gamma_{\uparrow}^H = \pi(1 - \cos \theta_i) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (54)$$

The non-abelian matrix Berry phase in Quantum Hall effect is originated due to the frustration offered by the magnetic field and the disorder of spins. In the absence of local frustration (latter) this complexity of connection will be removed. We would like to mention that spin echo between two single qubit has the equivalence of RVB state in FQHE and topological quantum computation with BP is responsible for the formation of higher states considering the Hall qubit at $\nu = 1$ as a building block of any QHE state.

4 Discussion

In this paper we have studied the Physics behind the singlet state entangled by the two qubits where one is rotating in the field of the other with the Berry phase only. This image of spin echo has been reflected in the field of Quantum Hall effect. The Hall state for the lowest Landau level at $\nu = 1$ is highly frustrated. They are the singlet states identified as the Hall qubit, the building block of other higher IQHE/FQHE states at different filling factors. These states have matrix Berry phase which are responsible for pumped charge flow. In other words the Berry phase acts as a local order parameter of singlet states. Further we pointed out that the antisymmetric nature of $\nu = 1/m$ FQHE states depend on their acquired Berry phase. Since these spin singlet states forming a RVB gas is equivalent to fractional quantum Hall fluid, the description of background Physics through quantum computation will be of ample interest. We will proceed to study the hierarchies of FQHE in the light of quantum communication in the future.

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References

- [1] J.Bell, Physics **1**, 95(1964); Rev.Mod.Phys.**38**,447 (1966).
- [2] M.A.Nielse and I.L.Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge (2000).
- [3] J.Preskill;Lecture Notes for Physics 219: Quantum Computation (2004).
- [4] M.V.Berry, Proc.R.Soc.London **A392**,45 (1984).
- [5] V.Vedral, Cent. Eur. J. Phys.**2**, 289 (2003);quant-ph/0505029.
V.Vedral,quantum-ph/0212133.
- [6] A.Y.Kitaev; Ann. Phys. **303**,2 (2003).
- [7] D.Banerjee; Phys. Rev.-**B58**,4666 (1998).
- [8] D.Banerjee and P.Bandyopadhyay, Physica Scripta, **73**,571(2006), ICTP preprint.
- [9] P.Bandyopadhyay and K.Hajra, J. math. Phys.**28**, 711 (1987).
- [10] P.Bandyopadhyay; Int.J.Mod.Phys.**A4**, 4449 (1989).
- [11] D.Banerjee and P.Bandyopadhyay;Nuovo Cimneto,**113**(1998)921; D.Banerjee; Fort.der Physik **44** (1996) 323
- [12] D.Banerjee;"The Berry phase in frustrated spin glass",ICTP preprint and communicated to "Euro Physics Journals-B".
- [13] N.Y.Hwang,S.C.Kim, P.S.Park and S.R.Eric Yang;arXiv; cond-mat/0706.0947.
- [14] A.Ekert, M. Ericsson, P.Hayden, H. Inamori, J. A. Jones, D.K.L.Oi and V. Vedral,arXiv: quantum-ph/0004015.
- [15] R.A.Bertlmann, K. Durstberger, Y. Hasegawa and B. C. Heismayr; Phys. Rev.-**A69**, 032112 (2004).
- [16] B.Basu ; arXiv: quant-ph/0602089.
- [17] J.K. Jain and R.K.Kamilla in "Composite Fermions" edited by Olle Heironen(World Scientific, New York, 1998.)
- [18] V.Kalmeyer and R.B.Laughlin; Phys.Rev.Lett. **59**, 2095 (1987).
- [19] D.Banerjee and P.Bandyopadhyay; Mod.Phys.Lett. **B8**, 1643, (1994).